Game Theory

Lecture 0: Logistics and Motivation

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Course textbooks

- Bonanno, G. (2024). *Game Theory (3rd ed.)*. University of California, Davis. Received from: GT Book
- Axelrod, R. (1984). *The Evolution of Cooperation*. Basic Books. Received from: Axelrod Article
- Nisan, N., Roughgarden, T., Tardos, É., & Vazirani, V. V. (2007). *Algorithmic Game Theory*. Cambridge University Press. Received from: AGT Book
- Myerson, R. B. (1991). Game Theory: Analysis of Conflict. Harvard University Press. Received from: GT Book 2
- F. Christianos et al., *Multi-Agent Reinforcement Learning: Foundations and Modern Approaches*, 2023. Received from: MARL Book.pdf
- Shoham, Y., & Leyton-Brown, K. (2008). Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations. Cambridge University Press Received from: MARL Book.pdf
- `nashpy` documentation (readthedocs) Link: NashPy Docs

Outline

Logistics

Motivation

What is Game Theory?

Schedule

- **Lecture:** Wednesdays, 10:00 AM - 11:30 PM
- **Location:** South Building, Room 2-712

Lecture:

- László Gulyás (lgulyas@inf.elte.hu)

Practice:

- Tamás Takács (tamastheactual@inf.elte.hu)

Grading

Grading

- Final Lecture Score (LS) = Midterm 1 (50 points) + Midterm 2 (50 points)
- Final Practice Score (PS) = Assignment 1 (50 points) + Assignment 2 (50 points)
- Final Score (FS) = (LS + PS) / 2
- Final Exam (written):
  - Pass required on both LS and PS (individually) to attend the final exam
  - Pass/Fail exam to get the FS
  - Written exam from the lecture material

Grade Conversion

Final Score Range	Grade
> 85	5
75 - 85	4
65 - 74	3
40 - 64	2
< 40	Fail

Outline

Logistics

Motivation

What is Game Theory?

What is a Game?

What is a Game? - Questions

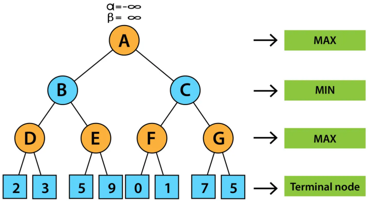
- A Boardgame?
- Chess?
- Video games? Sports?

What is a Game? - Alpha-Beta-Pruning

- A Boardgame?
- Chess?

Sure...

- Minimax algorithm
- Alpha-Beta cut



Familiar?

What is a Game? - Examples

Anything else?
Perhaps more general?

For example:

- Negotiating the price of a car
- Deciding where to sit in an empty classroom
- Online auctions

Can a traffic jam be a game? What about standing in line for coffee?

What is a Game? - Competition

- Competition? $\rightarrow$ How?
- Is every competition a game?
- When would competing not involve a "game"?

What is a Game? - Value

- Collecting
  - Money?
  - Points? Scores? $\rightarrow$ Artificial...
  - Value $\rightarrow$ Valuation
- Numerical (quantitative), or
- (Partial) ordering

What is a Game? - Outcomes

DECISION(s)

- Resulting in certain outcomes (numerical / ordered)

Can you think of situations with no clear "winner" or "loser"?

An interesting problem (Hotelling, 1929)

Two competing shops

- located along the length of a street
- selling the same good at the same price
- with customers spread equally along the street

Both shop owners want

- to position their shops to be where they will get most customers

Customers

- are indifferent between the shops,
- go to what is closest

Hotelling, 1929

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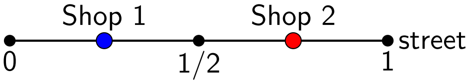
WHERE THE SHOPS WILL BE LOCATED?

Hotelling, 1929 - Visualization



- Where to put the shops?
- What if you could revise your decision?

Hotelling - Iteration

- What if the shops are located at the $\frac{1}{4}$ and $\frac{3}{4}$ of the street?

Hotelling - Iteration (Cont.)

- What if the shops are located at the $\frac{1}{4}$ and $\frac{3}{4}$ of the street?
- What if life goes on? Can shop owners revise their decisions?
- Over time, both move toward the center.

In your city, why are gas stations or coffee shops often clustered together?

Hotelling: Equilibrium

- When both shops are at $\frac{1}{2}$, neither can improve by moving alone
- **This is a Nash Equilibrium:** no one can do better by changing position, if the other stays put

Real-World Stories

- Tipping at the office canteen...
- Choosing when to post on social media for maximum likes
- Strategic pricing in supermarkets

What is common about these situations?

- Decisions
- Different values (valuations) of the outcomes

Anything else?

Outcome depends on others

- Decisions
- Different values (valuations) of outcomes
- **Outcome / Value of outcome depends on others**

(Value of) Outcome depends on others

Modeling others' behavior

$\longrightarrow$

I know that you know that I know that you know that...

Have you ever tried to guess someone else's move in a game or negotiation?

(Value of) Outcome depends on others - Behavioral assumptions

- Rational (maximizing / optimal) behavior
- What else?
  - Random?
  - “The wilderness of irrationality”
  - Learning or experience

(Value of) Outcome depends on others - Learning

- Behavioral assumptions
  - Rational (maximizing / optimal) behavior
  - Random?
  - "The wilderness of irrationality"
  - **Learning (approximating rationality)**
- Many real-life games involve repeated interaction and adaptation

Outline

Logistics

Motivation

What is Game Theory?

Decision Making - Three major elements

1. Who is in charge to make the decision? The decision maker (DM):
   - one or
   - more
2. What choices the DM has? Alternatives:
   - Finitely many (discrete problem), $A_1, A_2, ..., A_m$
   - Described by continuous variables (continuous problem), like $X = \{x | x \in \mathbb{R}^m, g(x) \leq 0\}$
3. What are the consequences of the decision?
   - Objective functions, $\phi_1, \phi_2, ..., \phi_n$.

Many cases - Decision scenarios

Number of DMs and number of objectives

	1 DM	Multiple DMs
1 objective (each)	single objective optimization	game
Multiple objectives	multiobjective optimization	Pareto game

One-off or repeated (iterated) games, etc.

Further examples

- Elections (voting strategies, alliances)
- Allocation problems (who gets what and why)
- Art Auctions (highest bid wins, second-price nuances)
- Public tenders (bidding for contracts, strategic pricing)
- Spectrum auctions (telecom companies, massive stakes)

Further examples (Cont.)

- Elections (voting strategies, alliances)
- Allocation problems (who gets what and why)
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Are these games? Can you find the "players" and "payoffs" in these situations?

Two Approaches to Game Theory

Bottom-Up

- Game $\longrightarrow$ (Equilibrium) Outcome

Top-Down

- Problem & (rational) DMs $\longrightarrow$ Game
  - *Implementation Theory* - design the game to get desired outcomes

Which approach seems more useful to you as a student or researcher?

History

- John von Neumann (1928): Early math foundations, minimax
- John Nash (1950-53): Equilibrium in games
- Nobel laureates Nash, Selten, Harsanyi (1996), Vickrey (auctions)
- Game theory now used in economics, computer science, biology, politics, and more

Terminology - Watch out

WARNING: Conflicting terminology

- Game Theory vs Reinforcement Learning
- The two textbooks used use different words for the same concepts!

Key concepts:

- DMs: players (agents)
- Decision alternatives: strategies (actions)
  - Probability distributions on alternatives: *mixed* strategies
- Objective functions: payoff functions (to be formally defined soon)

Discussion

Small groups:

Identify a real-life situation where "what you do" depends on "what others do", and sketch (in words) who are the players, what are their possible actions, and what might be the outcomes/payoffs.

Recap and What's Next

- Game theory investigates situations where the optimal choice depends on others' choices
- We've seen examples from games, economics, politics, daily life
- Next lecture: Formal framework for "Normal-Form Games," concrete examples, foundations for strategic thinking