

Game Theory

Lecture 1: Introduction to Normal-Form Games

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What is Game Theory?

Normal-Form Games

Classic 2x2 Games

Dominance and Iterated Elimination

Best Response Maps

Repeated Normal-Form Games and Policies over Histories

Examples and Exercises

Concluding Remarks

Course textbooks

- ▶ Bonanno, G. (2024). *Game Theory (3rd ed.)*. University of California, Davis. Received from: [GT Book](#)
- ▶ Axelrod, R. (1984). *The Evolution of Cooperation*. Basic Books. Received from: [Axelrod Article](#)
- ▶ Nisan, N., Roughgarden, T., Tardos, É., & Vazirani, V. V. (2007). *Algorithmic Game Theory*. Cambridge University Press. Received from: [AGT Book](#)
- ▶ Myerson, R. B. (1991). *Game Theory: Analysis of Conflict*. Harvard University Press. Received from: [GT Book 2](#)
- ▶ F. Christianos et al., *Multi-Agent Reinforcement Learning: Foundations and Modern Approaches*, 2023. Received from: [MARL Book.pdf](#)
- ▶ Shoham, Y., & Leyton-Brown, K. (2008). *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press
Received from: [MARL Book.pdf](#)
- ▶ nashpy documentation (readthedocs) Link: [NashPy Docs](#)

Previously on Lecture 0

- ▶ Defined “game” in broad sense: boardgames, auctions, negotiations, and everyday life
- ▶ Motivation from real-world scenarios (Hotelling’s shops, tipping, elections, resource allocation)
- ▶ Key features:
 - ▶ Players: Who makes decisions?
 - ▶ Actions: What are the choices?
 - ▶ Payoffs: How outcomes are valued by participants
- ▶ Introduced the idea that outcomes often depend on others’ choices
- ▶ Discussed behavioral assumptions: rationality, randomness, learning, and real-world adaptation
- ▶ Saw how Game Theory formalizes strategic interactions to analyze and predict behavior

Lecture Overview

- ▶ What is Game Theory and what is a Game?
- ▶ Normal-Form Games (NFGs): definition and key notation
- ▶ Axes for classifying games
- ▶ Detailed classic 2x2 games: strategy, payoffs, interpretation
- ▶ Symmetry, transformations, and equivalence
- ▶ Dominance and iterated elimination
- ▶ Best response concept and visualizations
- ▶ Introduction to repeated games and policies
- ▶ Practical Exercise

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Game Theory

Game theory studies how strategic agents interact, how their choices affect each other, and how to analyze their outcomes.

- ▶ Predict choices in economic, social, biological systems
- ▶ Design algorithms/protocols in computer science, cryptography, and networks
- ▶ Analyze stability, efficiency, and robustness

Where do you see strategic interaction outside of games?

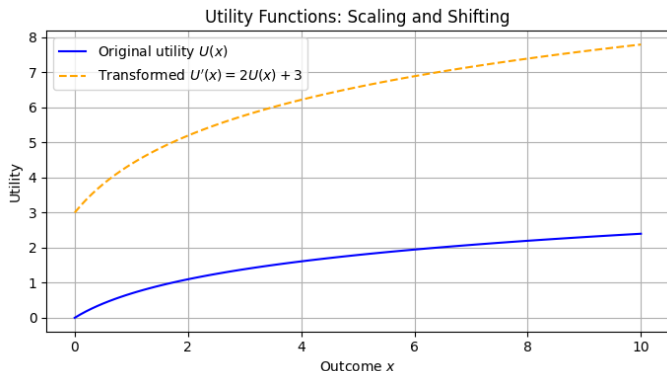
What is a Game?

A normal-form game is a formal structure for interactive decision-making.

- ▶ **Players:** $N = \{1, \dots, n\}$ (list the players)
- ▶ **Action sets:** A_i for each player i (choices available)
- ▶ **Outcomes:** Joint action $a = (a_1, \dots, a_n) \in A = \prod_i A_i$
- ▶ **Payoffs:** $R_i(a)$ is how much player i values outcome a

Preferences and Utilities

- ▶ Preferences are formalized by binary relations: $x \succeq_i y$ if player i likes x at least as much as y .
- ▶ Utility functions U_i map outcomes to numbers that capture this ordering.
- ▶ Utilities are only unique up to positive affine transformation: for any U , $U'(x) = a + bU(x)$ with $b > 0$ represents the same preferences.
- ▶ **Ordinal:** Order is all that matters (rankings).
- ▶ **Cardinal:** The magnitude of difference is meaningful (risk, expectation).



Information and Timing

- ▶ **Simultaneous move (normal-form):** All act at once, unaware of others' choices.
- ▶ **Sequential move (extensive-form):** Players can observe prior moves.
- ▶ **Complete info:** All payoffs and choices are known.
- ▶ **Incomplete info:** Hidden actions, private payoffs, uncertainty.
- ▶ **Perfect monitoring vs noisy monitoring:** Is everyone watching?

This lecture assumes simultaneous moves and complete information in the one-shot model.

Is chess a simultaneous or sequential move game? What about email negotiations or public auctions?

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Formal Definition

A normal-form game (NFG) is $(N, \{A_i\}_{i \in N}, \{R_i\}_{i \in N})$ with finite N and finite action sets A_i :

- ▶ N : players
- ▶ A_i : finite actions
- ▶ $R_i : A \rightarrow \mathbb{R}$: payoff

Play: All players choose $a_i \in A_i$ at once; tuple $a = (a_1, \dots, a_n)$ yields payoffs $R_i(a)$.

Notation Summary

- ▶ Players: N , $i \in N$
- ▶ Actions: A_i , joint space $A = \prod_i A_i$
 - ▶ In other terminology: strategies
- ▶ Joint action: $a = (a_1, \dots, a_n)$
- ▶ Payoffs: $R_i : A \rightarrow \mathbb{R}$
- ▶ Strategy: distribution π_i over A_i with $\sum_{a_i} \pi_i(a_i) = 1$
 - ▶ In other terminology: *mixed* strategy

In game theory, “strategy” can mean a choice at a single point (normal form), or more complex rules (in repeated/sequential games).

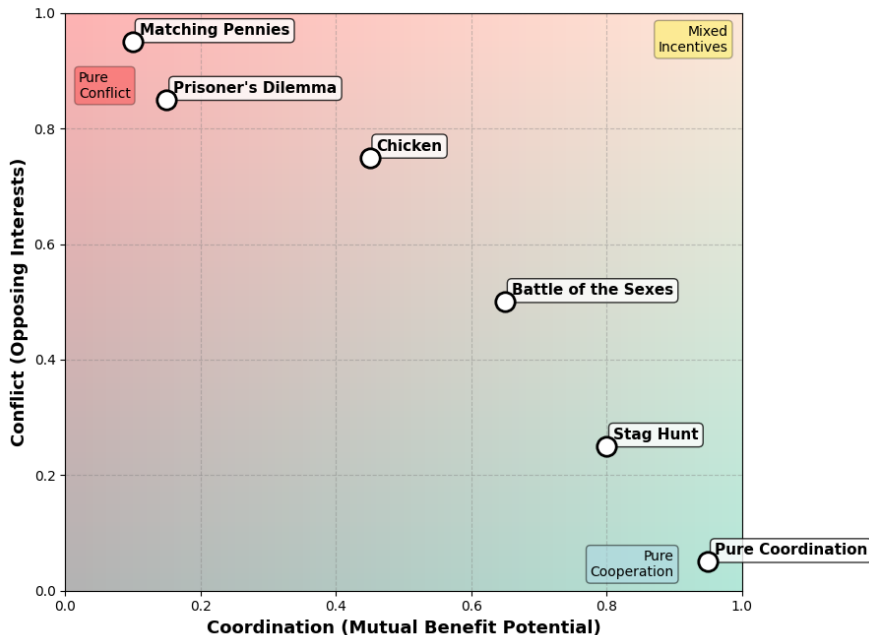
Classification Axes

- ▶ **Sum structure:** zero-sum (one's gain = other's loss), constant-sum, general-sum, common-payoff.
- ▶ **Symmetry:** Does switching labels for players and their actions leave payoffs unchanged?
- ▶ **Potential games (preview):** Is there a global potential function for which every unilateral move's payoff matches its effect on that function?
- ▶ **Risk dominance / payoff dominance** (especially for coordination/cooperation games): robustness vs social efficiency.

We use these lenses to interpret examples.

Coordination-Conflict Spectrum

Classic Games on the Coordination-Conflict Spectrum



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More on Classification

Feature	Example
Zero-sum	Matching Pennies, RPS
General-sum	Prisoner's Dilemma, Coordination
Common-payoff	Pure coordination, group planning
Symmetric	RPS, Battle of the Sexes (with swap)
Asymmetric	"Game of Pigs", Stackelberg Duopoly

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Matrix Representation and Exercise

Explain the structure of 2-player games as payoff matrices, and offer:

Table:

	C1	C2
R1	(r_1, r_2)	(r_1, r_2)
R2	(r_1, r_2)	(r_1, r_2)

- Row player chooses row, column player chooses column. Each cell gives both payoffs.

Draw a small matrix for “Choosing a movie with a friend” scenario (2 choices each).

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Normal-Form Game: Formal Definition

A normal-form game (NFG) is $(N, \{A_i\}_{i \in N}, \{R_i\}_{i \in N})$ with finite N and finite action sets A_i :

- ▶ N : players
- ▶ A_i : finite actions
- ▶ $R_i : A \rightarrow \mathbb{R}$: payoff

Two-player games are often displayed as matrices with entries (r_1, r_2)

Matrix Representation

	C1	C2
R1	(r_1, r_2)	(r_1, r_2)
R2	(r_1, r_2)	(r_1, r_2)

Row player chooses row, column player chooses column. Each cell gives both payoffs.

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1. Battle of the Sexes

Real-Life Story:

A couple wants to spend the evening together. She prefers ballet, he prefers football. Both prefer being together over being apart, but each has their own preference.

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1. Battle of the Sexes: The Game

	Ballet	Football
Ballet	(2, 1)	(0, 0)
Football	(0, 0)	(1, 2)

Best Response Analysis:

► Row player (She):

- If Column plays Ballet \rightarrow Row's best: Ballet ($2 > 0$)
- If Column plays Football \rightarrow Row's best: Football ($1 > 0$)

► Column player (He):

- If Row plays Ballet \rightarrow Column's best: Ballet ($1 > 0$)
- If Row plays Football \rightarrow Column's best: Football ($2 > 0$)

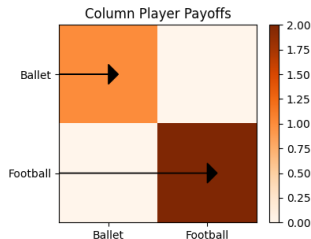
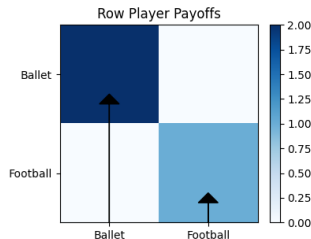
Mutual Best Responses: (Ballet, Ballet) and (Football, Football)

1. Battle of the Sexes: Discussion

	Ballet	Football
Ballet	(2,1)	(0,0)
Football	(0,0)	(1,2)

- Coordination with conflicting preferences
- Two pure profiles are efficient, each favors a different player

Battle of the Sexes: Payoffs and Best Responses



1. Battle of the Sexes: Questions

	Ballet	Football
Ballet	(2,1)	(0,0)
Football	(0,0)	(1,2)

1. If you're Row, how do you decide what to choose without communication?
2. Is there a "fair" outcome? Who gets what they want in each mutual BR?
3. What real-life mechanisms help people coordinate here?

2. Coordination Game

Real-Life Story:

Two friends want to watch something together on streaming. Alice slightly prefers sports, Bob slightly prefers comedy. Miscoordination (different choices) means they can't enjoy it together.

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2. Coordination Game: The Game

	Bob: Sport	Bob: Comedy
Alice: Sport	(3, 2)	(1, 1)
Alice: Comedy	(0, 0)	(2, 3)

Best Response Analysis:

▶ Alice (Row):

- ▶ If Bob plays Sport \rightarrow Alice's best: Sport ($3 > 0$)
- ▶ If Bob plays Comedy \rightarrow Alice's best: Comedy ($2 > 1$)

▶ Bob (Column):

- ▶ If Alice plays Sport \rightarrow Bob's best: Sport ($2 > 0$)
- ▶ If Alice plays Comedy \rightarrow Bob's best: Comedy ($3 > 1$)

Mutual Best Responses: (Sport, Sport) and (Comedy, Comedy)

2. Coordination Game: Discussion

	Bob: Sport	Bob: Comedy
Alice: Sport	(3, 2)	(1, 1)
Alice: Comedy	(0, 0)	(2, 3)

- ▶ Interests partially aligned
- ▶ Two efficient action profiles exist but differ by distribution

```
import nashpy as nash, numpy as np
A = np.array([[3,1],[0,2]])
B = np.array([[2,1],[0,3]])
game = nash.Game(A,B)

eqs = list(game.support_enumeration())
for i, (sigma_a, sigma_b) in enumerate(eqs, 1):
    payA, payB = game[sigma_a, sigma_b]
    print(f"Eq {i}: Alice {sigma_a}, Bob {sigma_b} \\
        -> payoffs ({payA:.3f}, {payB:.3f})")
```

2. Coordination Game: Questions

	Bob: Sport	Bob: Comedy
Alice: Sport	(3, 2)	(1, 1)
Alice: Comedy	(0, 0)	(2, 3)

1. Is (Sport, Sport) better than (Comedy, Comedy)? For whom?
2. What coordination mechanisms exist in real life?
3. If you played this repeatedly, could you establish a pattern?

3. Stag Hunt

Real-Life Story:

Two hunters can cooperate to hunt a stag (high reward, requires both) or individually hunt hare (lower reward, guaranteed). Based on Jean-Jacques Rousseau's philosophy.

3. Stag Hunt: The Game

	Stag	Hare
Stag	(3,3)	(0,2)
Hare	(2,0)	(2,2)

Best Response Analysis:

► Row player:

- If Column plays Stag \rightarrow Row's best: Stag ($3 > 2$)
- If Column plays Hare \rightarrow Row's best: Hare ($2 > 0$)

► Column player:

- If Row plays Stag \rightarrow Column's best: Stag ($3 > 0$)
- If Row plays Hare \rightarrow Column's best: Hare ($2 > 2$)

Mutual Best Responses: (Stag, Stag) and (Hare, Hare)

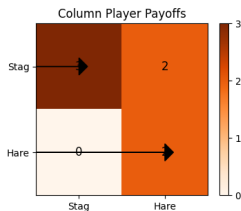
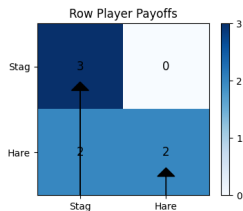
3. Stag Hunt: Discussion

	Stag	Hare
Stag	(3,3)	(0,2)
Hare	(2,0)	(2,2)

- ▶ Payoff-dominant vs risk-dominant equilibria
- ▶ Captures trust and assurance problems

Stag-Stag is payoff dominant (highest payoffs), but Hare-Hare is risk dominant (safer).

Stag Hunt: Payoffs and Best Responses



3. Stag Hunt: Questions

	Stag	Hare
Stag	(3,3)	(0,2)
Hare	(2,0)	(2,2)

1. If you don't trust your partner, what do you choose? Why?
2. What would change if you could communicate beforehand?
3. Can you think of international relations examples (climate change, arms control)?
4. How might repeated interaction affect your choice?

4. Chicken (Hawk-Dove)

Real-Life Story:

Two drivers speed toward each other. The first to swerve “loses face” but avoids catastrophe. If neither swerves, disaster.

4. Chicken (Hawk-Dove): The Game

	Swerve	Straight
Swerve	(0,0)	(-1,1)
Straight	(1,-1)	(-10,-10)

Best Response Analysis:

► Row player:

- If Column Swerves \rightarrow Row's best: Straight ($1 > 0$)
- If Column goes Straight \rightarrow Row's best: Swerve ($-1 > -10$)

► Column player:

- If Row Swerves \rightarrow Column's best: Straight ($1 > 0$)
- If Row goes Straight \rightarrow Column's best: Swerve ($-1 > -10$)

Mutual Best Responses: (Swerve, Straight) and (Straight, Swerve)

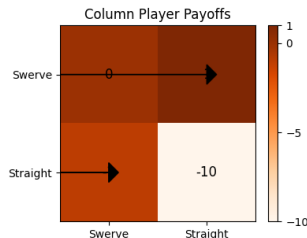
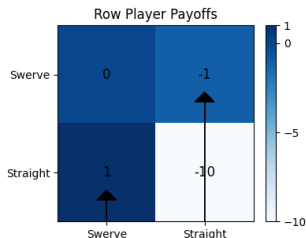
BRs are *anti-diagonal*: you want to do the opposite of opponent!

4. Chicken (Hawk-Dove): Discussion

	Swerve	Straight
Swerve	(0,0)	(-1,1)
Straight	(1,-1)	(-10,-10)

- Strategic aggression and brinkmanship
- Mutual stubbornness is catastrophic

Chicken (Hawk-Dove): Payoffs and Best Responses



4. Chicken (Hawk-Dove): Questions

	Swerve	Straight
Swerve	(0,0)	(-1,1)
Straight	(1,-1)	(-10,-10)

1. Why is (Straight, Straight) so bad? Is it ever rational?
2. What gives a player credibility in “not swerving”?
3. Can you commit to a strategy before your opponent? How does that help?
4. Compare this to Prisoner’s Dilemma. What’s different about the incentives?

5. Matching Pennies (Zero-Sum)

Real-Life Story:

Two players simultaneously show either heads or tails of a coin. If they match, Row wins. If they differ, Column wins.

Examples from real life:

- ▶ Penalty kicks in soccer (keeper vs striker)
- ▶ Rock-paper-scissors
- ▶ Hide-and-seek, poker bluffing

5. Matching Pennies: The Game

	H	T
H	(1,-1)	(-1,1)
T	(-1,1)	(1,-1)

Best Response Analysis:

► Row player:

- If Column plays H \rightarrow Row's best: H ($1 > -1$)
- If Column plays T \rightarrow Row's best: T ($1 > -1$)

► Column player:

- If Row plays H \rightarrow Column's best: T ($1 > -1$)
- If Row plays T \rightarrow Column's best: H ($1 > -1$)

Mutual Best Responses: NONE! BRs cycle forever.

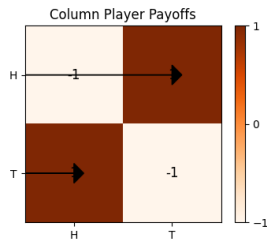
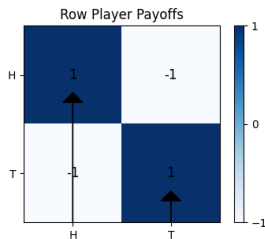
This is a **zero-sum** game (payoffs sum to zero in every cell).

5. Matching Pennies: Discussion

	H	T
H	(1,-1)	(-1,1)
T	(-1,1)	(1,-1)

- ▶ Zero sum
- ▶ Pure best responses cycle
- ▶ Requires mixing for stability (details later in the course)

Matching Pennies: Payoffs and Best Responses



5. Matching Pennies: NashPy

	H	T
H	(1,-1)	(-1,1)
T	(-1,1)	(1,-1)

```
import nashpy as nash
import numpy as np

A = np.array([[ 1, -1],[-1,  1]])
B = np.array([[-1,  1],[ 1, -1]])

game = nash.Game(A, B)
for sa, sb in game.support_enumeration():
    payA, payB = game[sa, sb]
    print(f"Row {sa}, Col {sb} -> payoffs ({payA:.3f}, {payB:.3f})")

Row [0.5 0.5], Col [0.5 0.5] -> payoffs (0.000, 0.000)
```

5. Matching Pennies: Questions

	H	T
H	(1,-1)	(-1,1)
T	(-1,1)	(1,-1)

1. Can you ever find a “stable” pure strategy pair? Why not?
2. What does this tell us about predictability?
3. How would you play this game?
4. Is this game fair?

6. Rock–Paper–Scissors (Zero-Sum)

	R	P	S
R	0	-1	1
P	1	0	-1
S	-1	1	0

Best Response Analysis:

► Row player:

- If Column plays Rock \rightarrow Row's best: Paper ($1 > 0, -1$)
- If Column plays Paper \rightarrow Row's best: Scissors ($1 > 0, -1$)
- If Column plays Scissors \rightarrow Row's best: Rock ($1 > 0, -1$)

► Column player: (By symmetry, analogous)

Mutual Best Responses: NONE! Cyclic dominance: Rock $<$ Paper $<$ Scissors $<$ Rock.

6. Rock–Paper–Scissors: Discussion

	R	P	S
R	0	-1	1
P	1	0	-1
S	-1	1	0

- ▶ Cyclic dominance
- ▶ No pure equilibrium
- ▶ No pure strategy is safe
- ▶ Each action beats one and loses to one

Modern examples:

- ▶ Evolutionary biology (species competition cycles)
- ▶ Market competition with cyclic advantages
- ▶ Combat strategies in games (tank-infantry-artillery cycles)

6. Rock–Paper–Scissors: Questions

	R	P	S
R	0	-1	1
P	1	0	-1
S	-1	1	0

1. Why can't you find a safe pure strategy?
2. What would happen if you always played Rock?
3. How is this different from Matching Pennies?
4. Can you predict what a human opponent will do?

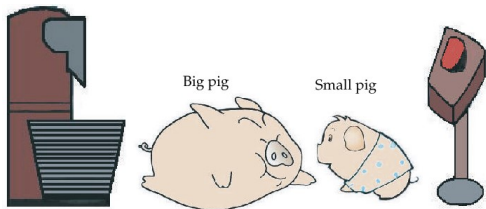
Symmetry and Relabeling

- ▶ **Symmetric game:** Swapping player identities leaves payoffs unchanged
 - ▶ Examples: Matching Pennies, RPS, symmetric Prisoner's Dilemma
- ▶ **Asymmetric game:** Players have fundamentally different roles
 - ▶ Examples: Game of Pigs, Battle of the Sexes
- ▶ Many population models use symmetric games
- ▶ Relabeling actions does not change strategic structure

For each game above, determine if it's symmetric or asymmetric. Justify your answer.

Asymmetric Game: Game of Pigs

- ▶ Two pigs: big and small
- ▶ A lever to press for food
- ▶ Food appears on the other end of pen
- ▶ There is a cost for pressing (energy)



	Big: Press	Big: Wait
Small: Press	$(4, 2)$	$(2, 3)$
Small: Wait	$(6, -1)$	$(0, 0)$

Game of Bigs: BR Analysis

	Big: Press	Big: Wait
Small: Press	(4, 2)	(2, 3)
Small: Wait	(6, -1)	(0, 0)

Small Pig:

- ▶ If Big presses: Press (4) vs Wait (6) \rightarrow Wait is better
- ▶ If Big waits: Press (2) vs Wait (0) \rightarrow Press is better
- ▶ **No dominant strategy**

Big Pig:

- ▶ If Small presses: Press (2) vs Wait (3) \rightarrow Wait is better
- ▶ If Small waits: Press (-1) vs Wait (0) \rightarrow Wait is better
- ▶ **Dominant strategy for Big Pig:** Wait

Mutual Best Response: (Small: Press, Big: Wait) \rightarrow (2, 3)

Game of Bigs: Questions

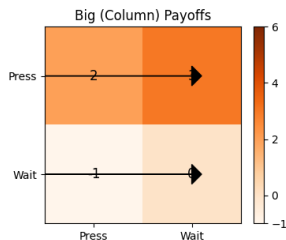
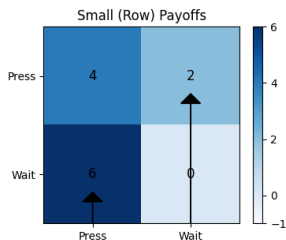
	Big: Press	Big: Wait
Small: Press	(4, 2)	(2, 3)
Small: Wait	(6, -1)	(0, 0)

1. Why does the small pig press when the big pig waits, but wait when the big pig presses?
2. Is the equilibrium (Press, Wait) fair? How might you adjust the numbers to share benefits more evenly?
3. Can you think of a workplace or social situation where one person does all the work and the other benefits?
4. What happens if the big pig's pressing cost increases further? When would Big no longer have Wait as a dominant strategy?

Game of Bigs: BRs

	Big: Press	Big: Wait
Small: Press	(4, 2)	(2, 3)
Small: Wait	(6, -1)	(0, 0)

Game of Pigs: Payoffs and Best Responses



Transformations and Equivalence

- ▶ Adding a constant to R_i does not affect best responses
- ▶ Positive affine transformations preserve argmax structure
- ▶ Constant-sum vs zero-sum conversions

Interpersonal utility comparisons are not meaningful without common scale assumptions.

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Dominance: Definitions

- ▶ Action a_i **strictly dominates** b_i if $R_i(a_i, a_{-i}) > R_i(b_i, a_{-i})$ for all a_{-i}
- ▶ **Weak dominance:** $R_i(a_i, a_{-i}) \geq R_i(b_i, a_{-i})$ for all a_{-i} , and strictly better for some a_{-i}
- ▶ Dominated actions are never rational to play

For the Game of Pigs, identify any strictly or weakly dominated strategies for each player.

Iterated Elimination of Strictly Dominated Strategies (IESDS)

- ▶ Remove strictly dominated actions for any player
- ▶ Repeat on the reduced game
- ▶ Order of elimination does not affect the final reduced game under strict dominance
- ▶ **Caveat:** Weak dominance can depend on elimination order

Apply IESDS to the Prisoner's Dilemma and Stag Hunt matrices below.

Dominance Practice: Prisoner's Dilemma

	Bob: C	Bob: D
Alice: C	(3,3)	(0,5)
Alice: D	(5,0)	(1,1)

- ▶ For each player, D strictly dominates C
- ▶ IESDS yields unique outcome (D,D)

Verify inequalities cell by cell.

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Dominance Practice: Stag Hunt

	Stag	Hare
Stag	(3,3)	(0,2)
Hare	(2,0)	(2,2)

- ▶ No strict dominance
- ▶ Context for risk vs payoff dominance

Identify safe vs efficient outcomes.

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Best Response: Definition

Given opponents' actions a_{-i} , the best response set is

$$BR_i(a_{-i}) = \arg \max_{a_i \in A_i} R_i(a_i, a_{-i})$$

A **Best Response Correspondence** maps opponents' actions to a set of optimal actions.

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Computing Best Responses by Inspection

- ▶ Fix a column (opponent action)
- ▶ Choose row that maximizes your payoff in that column
- ▶ Repeat for each column

This yields arrows or underlines in payoff matrices to visualize BRs.

For the Game of Pigs, underline the best responses for each player in the matrix.

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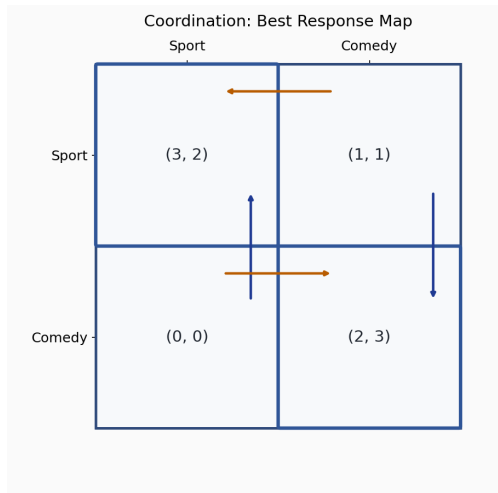
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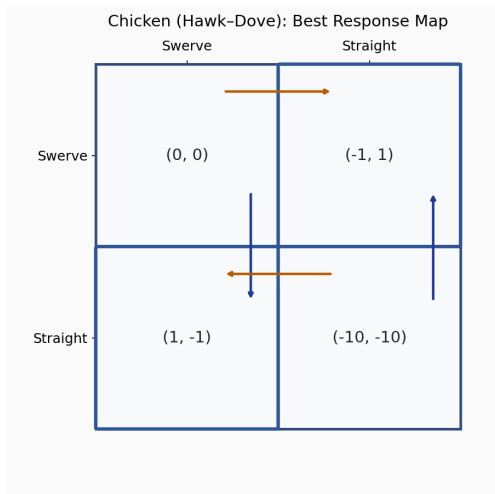
Best Response Maps: Coordination Game



Interpretation: both players best respond by matching

- Two mutual best responses exist.

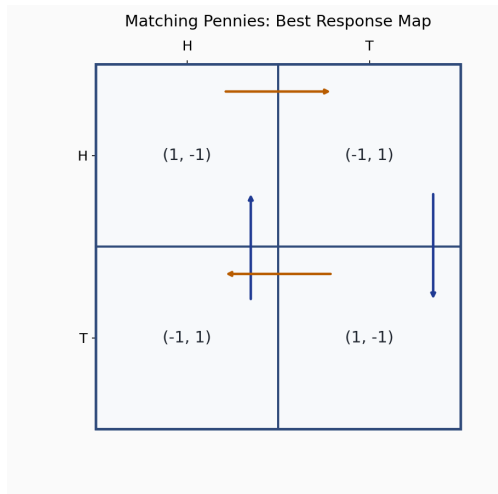
Best Response Maps: Chicken



Interpretation: anti-coordination incentives

- Two mutual best responses exist, diagonal entries are not both BRs.

Best response Maps: Matching Pennies



Interpretation: cycling best responses

- No mutual best response in pure strategies.

Payoff Normalization and Scaling

- ▶ Scaling by positive factor preserves BR and dominance
- ▶ Shifting by constant preserves comparisons within a player's payoffs
- ▶ Only preference ordering matters for pure-strategy reasoning

Rescale Battle of the Sexes and check BR structure.

	Ballet	Football
Ballet	(2,1)	(0,0)
Football	(0,0)	(1,2)

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Battle of Sexes Normalized

Rescaled (affine) payoffs using

- ▶ Row: $u' = 3u + 4$
- ▶ Column: $v' = 2v + 1$ (positive scaling + shift preserves BRs)

	Ballet	Football
Ballet	(10, 3)	(4, 1)
Football	(4, 1)	(7, 5)

Still a coordination game!

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Given base game $\Gamma = (N, \{A_i\}, \{R_i\})$, repeat for $t = 0, 1, \dots, T - 1$.

- ▶ History $h_t = (a^0, \dots, a^{t-1})$
- ▶ Strategy is a mapping from histories to actions or distributions
- ▶ Payoffs aggregated via average or discounting

Discounting and Aggregation

- ▶ Discounted return: $\sum_{t=0}^{\infty} \gamma^t r_i^t$, with $\gamma \in [0, 1)$
- ▶ Average reward: $\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} r_i^t$

Choice affects evaluation but not one-shot best responses

- ▶ Finite vs infinite games
- ▶ Uncertainty about game ending

Axelrod's Tournament (1980)

Tournament of Iterated Prisoners' Dilemma (IPD)

- ▶ Repeated PD
 - ▶ What is the dominant strategy in IPD?
- ▶ The success of Tit-for-Tat (TFT)

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Reputation and Memory in Repeated Games

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- ▶ Memory-1 strategies condition only on last round
- ▶ Examples: Tit-for-Tat, Grim Trigger, Pavlov (win-stay, lose-shift)
- ▶ Longer memory allows richer behavior

We do not cover learning rules today.

Examples of Repeated Strategies

Tit-for-Tat:

- ▶ Cooperate initially, then copy opponent

Grim Trigger:

- ▶ Cooperate until opponent defects once, then defect forever

Pavlov:

- ▶ Repeat previous action if you received high payoff, otherwise switch

Repeated PD Intuition

- ▶ With sufficient patience (γ high), cooperative paths can yield higher long-run payoffs
- ▶ Cooperation can be sustained by credible threat of future punishment

Formal results are covered later; focus now on interpreting incentives.

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Practice Example 1

Given the matrix below, underline the best responses for each player and identify any mutual best responses.

	L	R
U	(4,1)	(0,0)
D	(1,0)	(2,2)

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Practice Example 2

Classify the following game as zero-sum or general-sum and justify.

	L	R
U	(1,-1)	(-1,1)
D	(-1,1)	(1,-1)

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Practice Example 3

Is there any strictly dominated action for either player in this game?

	L	R
U	(2,2)	(0,3)
D	(3,0)	(1,1)

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Edge Cases and Modeling Cautions

- ▶ **Ties in payoffs:** If two or more actions yield the same payoff, a player may have multiple best responses. This can lead to multiple equilibria or indifference, and sometimes makes prediction harder.
- ▶ **Weak dominance:** Eliminating weakly dominated strategies can depend on the order of elimination. Sometimes, removing a weakly dominated strategy changes which other strategies are weakly dominated.
- ▶ **Non-generic games:** Some games have payoffs that are exactly equal for different actions, or have cycles in best response dynamics. These may require small perturbations or careful analysis to select equilibria.
- ▶ **Degeneracy:** In some games, many strategies are equally good, leading to a large set of best responses and possible equilibria.
- ▶ **Best response dynamics:** In potential games, best response dynamics always converge to a pure Nash equilibrium. In other games, they may cycle or get stuck.

Edge Cases and Modeling Cautions - Questions

1. Can you construct a matrix where a player is indifferent between two actions for some opponent move?
2. What happens if both players have multiple best responses at the same time?
3. Try to find a game where best response dynamics never settle down.

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- ▶ Reading matrices and best responses is the foundation for equilibrium concepts.
- ▶ Next week: best response dynamics, Nash equilibrium, and ε -Nash.
- ▶ Mixed strategies and expected payoffs arrive in two weeks.

Why do we care about algorithms in game theory? What kinds of real-world problems require computational solutions?

Summary

- ▶ Defined games and utilities, clarified normal-form representation.
- ▶ Surveyed classic 2x2 games and their incentives, with best response analysis.
- ▶ Introduced dominance, iterated elimination, and best response correspondences.
- ▶ Outlined repeated games and history-based strategies.
- ▶ Discussed edge cases, modeling cautions, and the importance of computational tools.

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