GAME THEORY

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Game Theory

Lecture 1

ELTE EÖTVÖS LORÁND UNIVERSITY

Introduction

1 Course Details

2 Motivation

3 What is GT?

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Course Textbooks

- Bonanno, G. (2024). ***Game Theory (3rd ed.)***. University of California, Davis. Received from: GT Book
- Axelrod, R. (1984). ***The Evolution of Cooperation***. Basic Books. Received from: Axelrod Article
- Nisan, N., Roughgarden, T., Tardos, É., & Vazirani, V. V. (2007). ***Algorithmic Game Theory***. Cambridge University Press. Received from: AGT Book
- Myerson, R. B. (1991). ***Game Theory: Analysis of Conflict.*** Harvard University Press. Received from: GT Book 2F.
- Christianos et al., ***Multi-Agent Reinforcement Learning: Foundations and Modern Approaches***, 2023. Received from: MARL Book.pdf
- Shoham, Y., & Leyton-Brown, K. (2008). ***Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations***. Cambridge University Press. Received from: MARL Book.pdf
- `***nashpy***` documentation (readthedocs). Link: NashPy Docs

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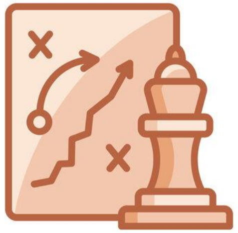
Schedule

Lecture:

- **Time**: Wednesdays, 10:00 AM - 11:30 PM
- **Location**: South Building, Room 2-502
- **Lecturer**: László Gulyás



Practice:

- **Time**: Wednesdays, 12:00 PM - 14:00 PM
- **Location**: South Building, Room 00-807
- **Lecturer**: Tamás Takács

Grading

- Final Lecture Score (**LS**) = Midterm 1 (50 points) + Midterm 2 (50 points)
- Final Practice Score (**PS**) = Assignment 1 (50 points) + Assignment 2 (50 points)
- Final Score (**FS**) = (**LS** + **PS**) / 2
- **Final Exam (written):**
  - Pass required on both **LS** and **PS** (individually) to attend the final exam
  - Pass/Fail exam to get the **FS**
  - Written exam from the lecture material
- Grade Conversion:

Final Score Range	Grade
> 85	5
$75 - 85$	4
$65 - 74$	3
$40 - 64$	2
< 40	$Fail$

Lecture 1

Introduction

What is a Game?

What is a Game?

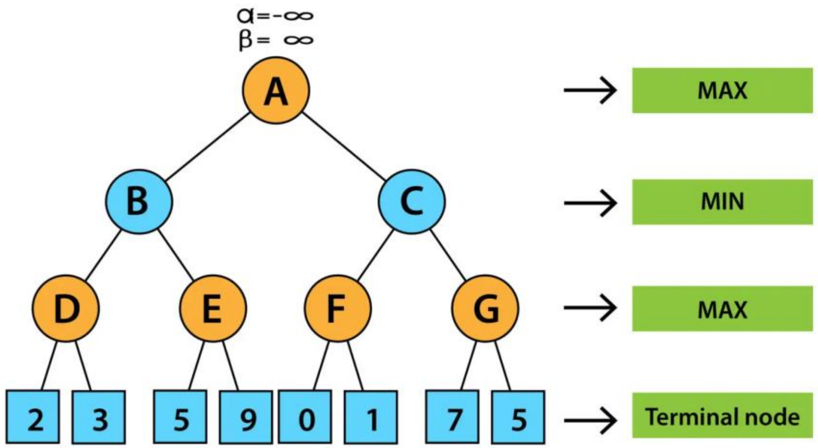
- A Boardgame?
- Chess?
- Video Games? Sports?

What is a Game?

- A Boardgame?
- Chess?

Sure...

- Minimax algorithm
- Alpha-Beta cut



Familiar?

What is a Game?

Anything else?
Perhaps more general?

For example:

- Negotiating the price of a car
- Deciding where to sit in an empty classroom
- Online auctions

Can a traffic jam be a game? What about standing in line for coffee?

What is a Game?

- **Competition?** → How?
- Is every **competition** a game?
- When would **competing** not involve a "game"?

What is a Game?

- **Collecting**
  - Money?
  - Points? Scores? → Artificial…
  - Value → Valuation
- **Numerical** (quantitative), or
- (Partial) **ordering**

What is a Game?

DECISION(s)

- Resulting in certain outcomes (numerical / ordered)

Can you think of situations with no clear "winner" or "loser"?

An interesting problem (Hotelling, 1929)

Two competing shops

- located along the length of a street
- selling the same good at the same price
- with customers spread equally along the street

Both shop owners want

- to position their shops to be where they will get most customers

Customers

- are indifferent between the shops
- go to what is closest

An interesting problem (Hotelling, 1929)

Two competing shops

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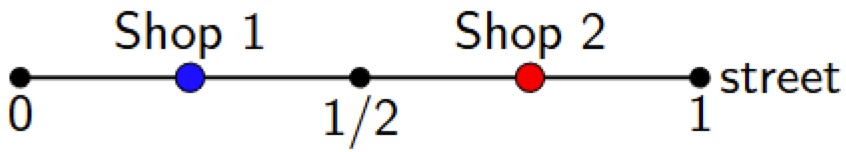
WHERE THE SHOPS WILL BE LOCATED?

An interesting problem (Hotelling, 1929)



- Where to put the shops?
- What if you could revise your decision?

Hotelling

- What if the shops are located at the $\frac{1}{4}$ and $\frac{3}{4}$ of the street?

Hotelling

- What if the shops are located at the $\frac{1}{4}$ and $\frac{3}{4}$ of the street?
- What if life goes on? Can shop owners revise their decisions?
- **Over time, both move toward the center.**

In your city, why are gas stations or coffee shops often clustered together?

Hotelling: Equilibrium

- When both shops are at $\frac{1}{2}$, neither can improve by moving alone
- **This is an Equilibrium:** no one can do better by changing position, if the other stays put

Real-World Stories

- Tipping at the office canteen…
- Choosing when to post on social media for maximum likes
- Strategic pricing in supermarkets

What is common about these situations?

- Decisions
- Different values (valuations) of the outcomes

Anything else?

Outcome depends on others

- Decisions
- Different values (valuations) of outcomes
- **Outcome / Value of outcome depends on others**

(Value of) Outcome depends on others

Modeling others' behavior ⟶

I know that you know that I know that you know that...

Have you ever tried to guess someone else's move in a game or negotiation?

Behavioral assumptions

- Rational (maximizing / optimal) behavior
- What else?
  - **Random?**
  - **"The wilderness of irrationality"**
  - **Learning or experience**

Learning

- Rational (maximizing / optimal) behavior
- What else?
  - **Random?**
  - **"The wilderness of irrationality“**
  - **Learning or experience**
- Many real-life games involve repeated interaction and adaptation

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Decision Making - Three major elements

1. **Who is in charge to make the decision? The decision maker (DM):**
   - one or
   - more

2. **What choices the DM has? Alternatives:**
   - Finitely many (discrete problem), $A_1, A_2, \dots, A_m$
   - Described by continuous variables (continuous problem), like $X = \{x \mid x \in R^m, g(x) \leq 0\}$

3. **What are the consequences of the decision?**
   - Objective functions, $\phi_1, \phi_2, \dots, \phi_n$.

Many Cases – Decision Scenarios

Number of DMs and number of objectives

	1 DM	Multiple DMs
1 objective (each)	Single objective optimization	Game
Multiple objectives	Multi-objective optimization	Pareto Game

One-off or repeated (iterated) games, etc.

Further examples

- Elections (voting strategies, alliances)
- Allocation problems (who gets what and why)
- Art Auctions (highest bid wins, second-price nuances)
- Public tenders (bidding for contracts, strategic pricing)
- Spectrum auctions (telecom companies, massive stakes)

Further examples

- Elections (voting strategies, alliances)
- Allocation problems (who gets what and why)
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Are these games?
Can you find the "players" and "payoffs" in these situations?

Two approaches to Game Theory

Bottom-Up

- Game $\longrightarrow$ (Equilibrium) Outcome

Top-Down

- Problem & (rational) DMs $\longrightarrow$ Game
    - **Implementation Theory** - design the game to get desired outcomes

Which approach seems more useful to you as a student or researcher?

History

- **John von Neumann** (1928): Early math foundations, minimax
- **John Nash** (1950-53): Equilibrium in games
- Nobel laureates Nash, Selten, Harsanyi (1996), Vickrey (auctions)
- Game theory now used in economics, computer science, biology, politics, and more

Terminology - Watch Out!

WARNING: Conflicting terminology

- Game Theory vs Reinforcement Learning
- The two textbooks used use different words for the same concepts!

Key concepts:

- DMs: players (agents)
- Decision alternatives: strategies (actions)
  - Probability distributions on alternatives: **mixed** strategies
- Objective functions: payoff functions (to be formally defined soon)

Terminology - Watch Out!

RL Term	GT Term	Description
environment	game	Model specifying the possible actions, observations, and rewards of agents
agent	player	Entity which makes decisions
reward	payoff, utility	Scalar value received by an agent/player after taking an action
policy	strategy	Function that maps observations to a probability distribution over actions
deterministic policy	pure strategy	Chooses a single action with probability 1
probabilistic policy	mixed strategy	Assigns a probability distribution over possible actions
joint policy	profile	Tuple of strategies or actions, one per agent/player
joint reward	payoff profile	Rewards received by all agents for a joint action
deterministic joint policy	pure strategy profile	All agents choose actions with probability 1
stochastic joint policy	mixed strategy profile	All agents may act probabilistically

Discussion

Small groups:

Identify a real-life situation where "what you do" depends on "what others do", and sketch (in words) who are the players, what are their possible actions, and what might be the outcomes/payoffs.

Summary

- Game theory investigates situations where the optimal choice depends on others' choices
- We've seen examples from games, economics, politics, daily life

That’s All for Today!